Collective coordinate analysis of inhomogeneous Nonlinear Klein-Gordon field theory

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Abstract

Two different sets of collective coordinate equations for solitary solutions of Nonlinear Klein-Gordon (NKG) model are introduced. The collective coordinate equations are derived using different approaches for adding the inhomogeneities as exrernal potentials to the soliton equation of motion. Interaction of the NKG field with a local inhomogeneity like a delta function potential wall and also delta function potential well is investigated using the presented collective coordinate equations and the results of two different models are compared. Most of the characters of the interaction are derived analytically. Analytical results are also compared with the results of numerical simulations.

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1 Introduction

Solitons are localized waves that have a nonzero energy density in a finite region of space and exponentially goes to zero as one moves away from this region.They appear in nonlinear classical field theories as stable and particle-like objects with finite mass and explicit structures.Therefore finding suitable methods for studying the soliton as a point particle help us to find a better perspective of the soliton behaviour. On the other hand, comparing the results of such these models with the results of direct numerical simulations determines the differences between solitons as point-like particles and real solitons. This topic is an interesting subject in nonlinear field theories [1]. Solitons appear in a nonlinear medium with a fine tuning between Nonlinear and dispersive effects. This means that they may be disappeared in the lack of this precise balance in the medium. It is clear that a real medium contains disorders and impurities. Therefore stability and propagation of solitons in such these media are of great interest because of its applications and also theoretical interests. In order to understanding the behaviour of nonlinear excitations in a disordered system, it is important to investigate the interaction of solitons with impurities.
Recently, some non-classical behaviours have been reported for solitons during the scattering from external potentials [2]. These potentials generally come from medium defects or impurities. Scattering of solitons of integrable systems from the potentials have been studied before[3]; but such this investigation for non integrable systems has not been reported yet. Therefore it is interesting to examine the methods of adding the potential to the NKG model as a non-integrable model and compare the outcomes with the results of integrable systems. These are strong motivations for investigation the scattering of NKG solitons from defects.

External potentials can be added to the equation of motion using different methods. One way is that adding an external potential to the equation of motion as perturbative terms [2, 3]. These effects also can be taken into account by making some parameters of the equation of motion to be function of space or time [4, 5]. Another way is adding an external potential to the field through the metric of background space-time [6, 7, 8]. This method can be used for models that their lagrangians are Lorentz invariant, such as Sine-Gordon model, ϕ^4 theory, CP^N model, NKG models and etc. In this paper we will focus on the behaviour of solitons of NKG and try to investigate the interaction of NKG solitons with defects by introducing two different analytical models.

Different types of the NKG model are important non-integrable models

which appear in some branches of science. These equations can be used for describing the particle dynamics in quantum field theory. Some of the other examples of NKG applications are: Discrete gap breathers in a diatomic chain [9], Dichotomous collective proton dynamics in ice [10], propagation and stability of relaxation modes in the Landau-Ginzburg model with dissipation[11] and Pion form factor [12]. Recently Wazwaz has proposed several localized solutions for NKG equations using "Tanh" method [13]. Solitons present different trajectories during the interaction with potentials. They can pass through or becomes trapped inside the potential after the interaction. This behaviour is very sensitive to the values of potential parameters in the model as well as to the initial conditions of scattered soliton. Most of the researches are in base of numerical studies in nonlinear field theories. Because such these systems are generally non-integrable. The collective coordinate approach helps us to find analytical equations for the evolution of localized solutions, clearly if one can construct such suitable variables. We will present two sets of collective coordinate variables extracted from different hypotheses [14]. These help us to talk about the validity of their results and predictions.
Therefore two models for the NKG field in an space dependent potential is presented in section 2. Both analytical models are introduced and will be solved in section 3. The results of two analytical models are compared in section 4 for potential barrier and potential well systems. In section 5 we will compare our analytical results with direct numerical solutions of the equations. Some conclusion and remarks will be presented in the section 6.

2 Two analytical models for NKG soliton-potential system

Model 1. Lagrangian of the NKG model in (1+1) dimensions is defined as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi) \tag{2.1}$$

where $U(\phi)$ is potential of the field which is defined by

$$U(\phi) = \lambda(x)\left(\frac{1}{2}\phi^2 - \frac{1}{2}\phi^4 + \frac{1}{8}\phi^6\right) \tag{2.2}$$

The equation of the motion from the Lagrangian (2.1) is

$$\partial_\mu\partial^\mu\phi + \lambda(x)\left(\phi - 2\phi^3 + \frac{3}{4}\phi^5\right) = 0 \tag{2.3}$$

Equation (2.3) has a soliton solution which can be written as [13]

$$\phi(x,t) = \left[1 + tanh\left(\sqrt{\lambda}\frac{x - X(t)}{\sqrt{1-\dot{X}^2}}\right)\right]^{1/2} \tag{2.4}$$

where $X(t) = x_0 - \dot{X}t$. x_0 and $\dot{X}$ are soliton initial position and its velocity respectively. It is a kink-like solution as figure 1 shows. We want to

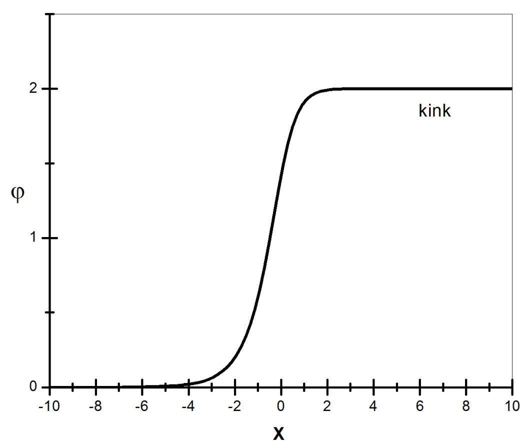

Figure 1: Kink-like solution of the NKG described by equation (2.3) for $x_0 = 0$ and $u = 0.5$ at $t = 0$.

investigate the behaviour of kink solution (2.4) during the interaction with an external potential. The external potential $V(x)$ can be added to the Lagrangian by taking a suitable space dependant function for the parameter λ as $\lambda(x) = 1 + V(x)$ [14, 15, 16]. In this approach we have added the potential directly to the equation through an extra term.
By inserting the solution (2.4) in the Lagrangian (2.1) and using adiabatic approximation [2] we have

$$\mathcal{L} = \frac{\left(\dot{X}^2 - 1\right) sech^4\left(x - X\right)}{8\left(1 + tanh\left(x - X\right)\right)}$$
$$-\frac{\lambda(x)}{2}\left[1 + tanh\left(x - X\right) - \left(1 + tanh\left(x - X\right)\right)^2 + \frac{\left(1 + tanh\left(x - X\right)\right)^3}{4}\right] \tag{2.5}$$

Model 2. The general form of the action in an arbitrary metric is

$$S = \int \mathcal{L}(\phi, \partial_\mu \phi)\sqrt{-g} d^n x dt \tag{2.6}$$

where "g" is the determinant of the metric $g^{\mu\nu}(x)$. Energy density of the system can be found by varying both the field and the metric [6]. For the Lagrangian of the form (2.1) the equation of motion becomes [8, 14]

$$\frac{1}{\sqrt{-g}}\left(\sqrt{-g}\partial_\mu \phi \partial^\mu \phi + \partial_\mu \phi \partial^\mu \sqrt{-g}\right) + \frac{\partial U(\phi)}{\partial \phi} = 0 \tag{2.7}$$

Space dependent potential can be added to the Lagrangian of the system by introducing a suitable nontrivial metric for the back ground space-time [6, 14]. In other words, the metric carries the information about the potential. In the presence of a weak potential V(x) the suitable metric is [6, 8, 14]

$$g_{\mu\nu}(x) \cong \begin{pmatrix} 1 + V(x) & 0 \\ 0 & -1 \end{pmatrix} \tag{2.8}$$

By inserting the solution (2.4) in the Lagrangian (2.1) with the potential (2.2) of NKG and using the metric (2.8), with adiabatic approximation [2, 3] we have

$$\mathcal{L} = \sqrt{1 + V(x)}\left(\left(1 + V(x)\right)\dot{X}^2 - 1\right)\frac{sech^4\left(\left(x - X\right)\right)}{8\left(1 + tanh\left(x - X\right)\right)}$$
$$-\frac{\sqrt{1 + V(x)}}{2}\left[1 + tanh\left(x - X\right) - \left(1 + tanh\left(x - X\right)\right)^2 + \frac{\left(1 + tanh\left(x - X\right)\right)^3}{4}\right] \tag{2.9}$$

For the weak potential $V(x)$ (2.9) becomes

$$\mathcal{L} \cong \left(\left(1 + \frac{3V(x)}{2}\right)\dot{X}^2 - \left(1 + \frac{V(x)}{2}\right)\right)\frac{sech^4\left(\left(x - X\right)\right)}{8\left(1 + tanh\left(x - X\right)\right)}$$
$$-\frac{1 + \frac{V(x)}{2}}{2}\left[1 + tanh\left(x - X\right) - \left(1 + tanh\left(x - X\right)\right)^2 + \frac{\left(1 + tanh\left(x - X\right)\right)^3}{4}\right] \tag{2.10}$$

3 Collective coordinate for two models

The Lagrangian density of the soliton is described by (2.5) in the model 1 and (2.10) in the model 2. These two equations are different in kinetic and also potential terms. We will compare them later. The soliton internal structure can be ommited by integrating the Lagrangian density (or Hamiltonian density) respect to the variable x. The integrated Lagrangian calls collective Lagrangian. After the integration, the soliton spears as a point-like particle, however the effects of its extended nature is reflected to the kinetic and also potential parts of the collective Lagrangian. The dynamics of the point-like particle can be described by equations which are derived using collective Lagrangian. It is interesting to compare the results of collective equations with direct numerical simulation of the main Lagrangian density. Let us derive the collective Lagrangian and the point-like particle equation of motion in two models.

Model 1. By integrating Lagrangian (2.5) over the variable x , $X(t)$ remains as a collective coordinate. If we take the potential $V(x) = \epsilon\delta(x)$ Collective lagrangian is derived from (2.5) as

$$L = \frac{1}{4}\dot{X}^2 + \frac{\epsilon}{2}\left[(tanh\,(X) - 1) + (1 - tanh\,(X))^2 - \frac{(1 - tanh\,(X))^3}{4}\right] - \frac{1}{2} \tag{3.11}$$

where $M_0 = \frac{1}{2}$ is the rest mass of the soliton in this model. The effective potential comes from equation (3.11) which is given by

$$U\,(\phi) = -\frac{\epsilon}{2}\left[tanh\,(X) - 1 + (1 - tanh\,(X))^2 - \frac{(1 - tanh\,(X))^3}{4}\right] + \frac{1}{2} \tag{3.12}$$

NKG solitons have a special and interesting situation which has not been observed in other fields. If we plot the effective potential as a function of collective position (X), we will find that it has a spatial shift respect to the origin. Figure 2(a) shows the effective potential as a function of position (X) for $\epsilon = 2$. Simulations also confirm this point for the NKG model. Figure 2(b) presents the shape of the potential barrier as seen by the soliton which is plotted using numerical calculation. It is an special characteristic for the NKG field theory. The source of this repugnance with the other fields needs more reconsideration.

The equation of motion for the variable $X(t)$ is derived from (3.11) as

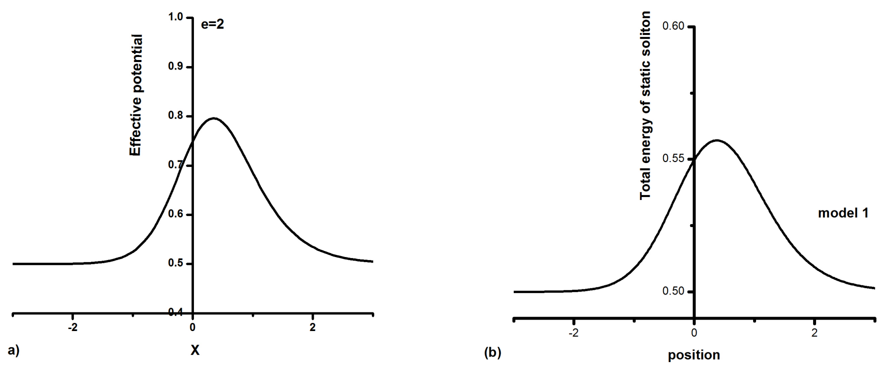

Figure 2: (a) Effective potential as a function of position with $\epsilon = 2$. (b) Potential barrier as seen by the soliton in the model 1 using numerical simulations.

$$\frac{1}{2}\ddot{X} - \frac{\epsilon}{2}sech^2(X)\left[1 - 2(1 - tanh(X)) + \frac{3(1 - tanh(X))^2}{4}\right] = 0 \quad (3.13)$$

We can define a collective force on the soliton if we look at the above equation as $F = M\ddot{X}$, where M is the rest mass of the soliton. Therefore we have

$$F = \frac{\epsilon}{2}sech^2(X)\left[\frac{3}{4}tanh^2(X) + \frac{1}{2}tanh(X) - \frac{1}{4}\right] \quad (3.14)$$

The above equation shows that the peak of the soliton moves under the influence of a complicated force which is a function of external potential and soliton position. Suppose that the soliton moves toward a potential barrier. Its velocity reduces because of affecting a repulsive force. After that soliton velocity increases when it goes away. Figures 3(a) and 3(b) show the force exerted by the potential well and barrier on the soliton for $\epsilon = -4$ and $\epsilon = 4$ respectively. This is in agreement with the observed behaviour for NKG model as shown in figure 2. These figures also show that the center of the force is not located in the origin.

Fortunately equation (3.13) has an exact solution for $\dot{X}$ as follows

$$\dot{X}^2 - \dot{X}_0{}^2 = \frac{\epsilon}{2}\left[tanh^3(X) + tanh^2(X) - tanh(X) - \tanh^3(X_0) - tanh^2(X_0) + tanh(X_0)\right] \quad (3.15)$$

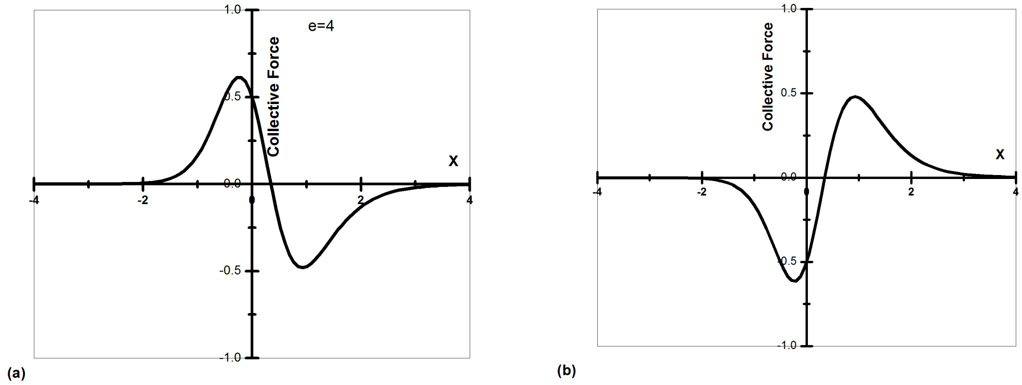

Figure 3: (a) The force on the soliton by a potential well with $\epsilon = -4$. (b) The force on the soliton by a barrier with $\epsilon = 4$.

where X_0 and $\dot{X}_0$ are soliton initial position and its initial velocity respectively. Some of the physical features of soliton-potential system can be found using equation (3.15). Collective energy is obtainable from Lagrangian (3.11) as follow

$$E = \frac{1}{4}\dot{X}^2 - \frac{\epsilon}{2}\left[(tanh\,(X) - 1) + (1 - tanh\,(X))^2 - \frac{(1 - tanh\,(X))^3}{4}\right] + \frac{1}{2} \tag{3.16}$$

It is the energy of a particle with the mass of $M_0 = \frac{1}{2}$ and velocity $\dot{X}$ which is moved under the influence of external effective potential. By substituting $\dot{X}$ from (3.15) in the equation (3.16), one can show that the energy of the system is a function of soliton initial conditions X_0 and $\dot{X}_0$ and therefore it is conserved.

Model 2. The above results can be calculated using the model 2. For the Lagrangian (2.10) $X(t)$ remains as a collective coordinate if we integrate (2.10) over the variable x. If we take the potential $V(x) = \epsilon\delta(x)$, the collective Lagrangian becomes

$$L = \left(\frac{1}{4} + \frac{3\epsilon sech^4\,(X)}{16\,(1 - tanh\,(X))}\right)\dot{X}^2$$
$$+\frac{\epsilon}{4}\left[(tanh\,(X) - 1) + (1 - tanh\,(X))^2 - \frac{(1 - tanh\,(X))^3}{4} - \frac{sech^4\,(X)}{4\,(1 - tanh\,(X))}\right] - \frac{1}{2} \tag{3.17}$$

The equation of motion for the variable X(t) is derived from (3.17)

$$\left(\frac{1}{2}+\frac{3\epsilon sech^4\left(X\right)}{8\left(1-tanh\left(X\right)\right)}\right)\ddot{X}$$
$$+\frac{\epsilon}{4}sech^2\left(X\right)\left[-\frac{3}{4}tanh^2\left(X\right)-\frac{1}{2}tanh\left(X\right)+\frac{1}{4}\right]\left(3\dot{X}^2+2\right)=0 \qquad (3.18)$$

The above equation describes the soliton trajectory which moves under the influence of a collective force. The collective force is a function of soliton position and also its velocity. $\dot{X}(t)$ can be calculated as a function of $X(t)$ by integrating (3.18) as follows

$$\frac{3\dot{X}^2+2}{3\dot{X_0}^2+2}=\frac{\frac{1}{2}+\frac{3\epsilon sech^4(X_0)}{8(1-tanh(X_0))}}{\frac{1}{2}+\frac{3\epsilon sech^4(X)}{8(1-tanh(X))}} \qquad (3.19)$$

Where X_0 and $\dot{X}_0$ are soliton initial position and its initial velocity respectively. The energy of the soliton in the presence of the potential $V(x)=\epsilon\delta(x)$ using model 2 becomes

$$E=\left(\frac{1}{4}+\frac{3\epsilon sech^4\left(X\right)}{16\left(1-tanh\left(X\right)\right)}\right)\dot{X}^2$$
$$-\frac{\epsilon}{4}\left[\left(tanh\left(X\right)-1\right)+\left(1-tanh\left(X\right)\right)^2-\frac{\left(1-tanh\left(X\right)\right)^3}{4}-\frac{sech^4\left(X\right)}{4\left(1-tanh\left(X\right)\right)}\right]+\frac{1}{2} \qquad (3.20)$$

Equation (3.20) shows that the rest mass is a function of soliton position in the model 2. This is the reason of some differences between two models which will be discussed in the next section. Some features of the soliton-potential dynamics are also studied using equations (3.19) and (3.20) analytically in the section 4 too.

4 Comparing the models

Potential barrier. There are two different trajectories for a soliton during the interaction with an effective potential barrier which are depend on the soliton initial conditions. A soliton with a low velocity reflects back from the barrier and a high velocity soliton climbs over the barrier and passes over it. So these two situations can be separated by a critical velocity. The total energy of the soliton-potential is conserved in both two models, as mentioned

before. Therefore we can find the critical velocity with a simple analysis from the energy of the soliton without any needed numerical simulations. Both equations (3.16) and (3.20) reduce to $E(X=\infty)=\frac{1}{4}\dot{X_0}^2+\frac{1}{2}$ when the soliton is far from the center of the delta-like potential which is located at the origin. It is the energy of a particle with the mass $M_0=\frac{1}{2}$ and velocity of $\dot{X_0}$. The energy of a soliton in the origin ($X=0$) comes from (3.16) and (3.20) for two models: $E_1(X=0)=\frac{1}{4}\dot{X_0}^2+\frac{\epsilon}{8}+\frac{1}{2}$ for the model 1 and $E_2(X=0)=\left(\frac{1}{4}+\frac{3\epsilon}{16}\right)\dot{X_0}^2+\frac{\epsilon}{8}+\frac{1}{2}$ for the model 2. The minimum energy of soliton in this position is $E=\frac{\epsilon}{8}+\frac{1}{2}$ for two models. On the other hand, a soliton which comes from the infinity with initial velocity v_c has the energy of $E=\frac{1}{4}{v_c}^2+\frac{1}{2}$. So it is easy to calculate the critical velocity of soliton by comparing the energy of the soliton at the origin and infinity. The critical velocity is obtained as $v_c=\sqrt{\frac{\epsilon}{2}}$ using both two models. The same result is derived by substituting $\dot{X}=0$, $\dot{X_0}=v_c$, $X_0=\infty$ and $X=0$ in (3.15) and (3.19).
Note that the critical velocity of the soliton depends on its initial position as well as its initial velocity. For a soliton which is located at some position like X_0 (which is not necessary infinity) the critical velocity will not be $v_c=\sqrt{\frac{\epsilon}{2}}$. So a soliton in the initial position X_0 with initial velocity of $\dot{X_0}$ has the critical initial velocity if its velocity becomes zero at the top of the barrier $X=0$. Consider a soliton with initial conditions of X_0 and $\dot{X_0}$. If we set $X=0$ and $\dot{X}=0$ in equations (3.15) and (3.19) then $v_c=\dot{X_0}$. Thus for the model 1 we have

$$v_c=\sqrt{-\frac{\epsilon}{2}\left(\tanh\left(X_0\right)-\tanh^2\left(X_0\right)-\tanh^3\left(X_0\right)\right)} \tag{4.21}$$

But the critical velocity in the model 2 becomes

$$v_c=\sqrt{\frac{2\epsilon\left(1-\tanh(X_0)-sech^4(X_0)\right)}{4-4\tanh(X_0)+3\epsilon sech^4(X_0)}} \tag{4.22}$$

Figure 4 shows critical velocity as a function of potential strength for $X_0=-1$ in two models. This figure shows that the model 2 predicts smaller critical velocity respect to the model 1 because of differences between rest masses of these models. It is clear that a soliton with great rest mass needs smaller velocity to reach the potential peak. Difference between the rest mass of the soliton in the models 1 and 2 can be calculated using (3.16) and (3.20) as

$$\triangle E_{kinetic}=\frac{3\epsilon sech^4(X)}{16\left(1-tanh(X)\right)}\dot{X}^2 \tag{4.23}$$

The above equation shows that two models predict equal rest mass at the infinity. But the difference between calculated rest mass in two models increases when the strength of the potential increases. Figure 4 shows this phenomenon explicitly. It is interesting to depict the critical velocity as a

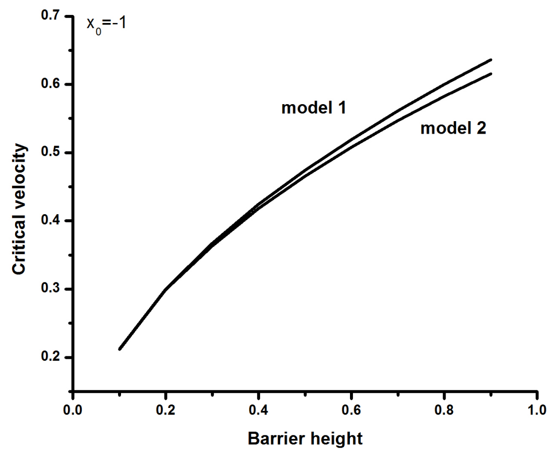

Figure 4: Critical velocity as a function of barrier height in both models for initial position $X_0 = -1$.

function of initial position. The critical velocity to pass over the potential has been demonstrated as a function of the initial position in the figure 5 for two models with $\epsilon = 0.5$. This figure shows there is a good agreement between these two models. For a soliton comes from infinity models are equal to each other as one can find from the figure 5. But there are some differences for a soliton moves from an initial posotion closer to the center of the potential. Figure 5 also demontrates that soliton needs lower initial velocity to pass over the barrier if it is closer to the center of the potential.

Soliton-well system. Let us consider a soliton which moves toward a frictionless potential well. This situation is very interesting because of some differences between point particle and a soliton in the potential well. A point particle falls in the well with an increasing velocity and reaches the bottom of the well with its maximum speed. After that, it will climb the well with a decreasing velocity and finally passes through the well. Its final velocity after the interaction is equal to its initial speed.

Potential well can be obtained by replacing ϵ with $-\epsilon$ in the equation (3.15).

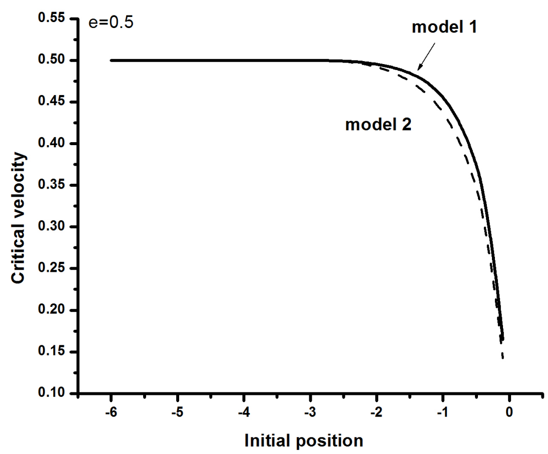

Figure 5: Critical velocity as a function of initial position in both models for $\epsilon = 0.5$.

The solution for the system in the model 1 is

$$\dot{X}^2 - \dot{X_0}^2 = -\frac{\epsilon}{2}\left[tanh^3(X) + tanh^2(X) - tanh(X) - tanh^3(X_0) - tanh^2(X_0) + tanh(X_0)\right] \tag{4.24}$$

Similary for the model 2 we have

$$\frac{3\dot{X}^2 + 2}{3\dot{X_0}^2 + 2} = \frac{\frac{1}{2} - \frac{3\epsilon sech^4(X_0)}{8(1 - tanh(X_0))}}{\frac{1}{2} - \frac{3\epsilon sech^4(X)}{8(1 - tanh(X))}} \tag{4.25}$$

We can define an escape velocity instead of critical velocity for a soliton-well system. The escape velocity is the minimum velocity for a soliton which can pass through the well. A soliton in an initial position X_0 reaches the infinity with a zero final velocity if its initial velocity is

$$\dot{X}_{escape1} = \sqrt{\frac{\epsilon}{2}\left(sech^2(X_0) - \tanh^3(X_0) + \tanh(X_0)\right)} \tag{4.26}$$

$$\dot{X}_{escape2} = \sqrt{\frac{2\epsilon sech^4(X_0)}{4 - 4\tanh(X_0) - 3\epsilon sech^4(X_0)}} \tag{4.27}$$

calculated using the models 1 and 2 respectively. In other words, a soliton which is located in the initial position X_0 can escape to infinity if its initial

velocity $\dot{X}_0$ is greater than the escape velocity $\dot{X}_{escape}$. Figures 6 and 7 show escape velocity from a potential well as a function of the well depth (figure 6) and soliton initial position (figure 7) using two models. A soliton in the model 1 needs lower escape velocity in comparson with the model 2 due to its bigger rest mass as one can find from the equation (4.23).

Consider a soliton which moves toward the potential well with an initial

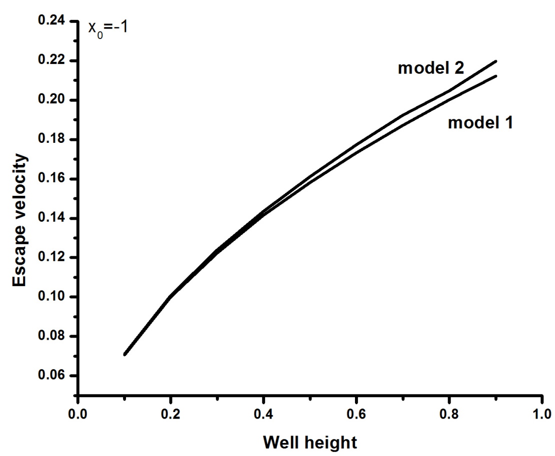

Figure 6: Escape velocity as a function of the well depth in both models for initial position $X_0 = -1$.

velocity $\dot{X}_0$ smaller than the escape velocity $\dot{X}_{escape}$. The soliton reaches a maximum distance X_{max} from the center of the potential with a zero velocity and then come back toward the center of the potential well. Therefore the soliton oscillates in the well with the amplitude of X_{max}. The required initial velocity to reach X_{max} is found from (4.25) for the model 2 as

$$\dot{X}_0 = \sqrt{\frac{2\,(1 - tanh\,(X_0))\,(4 - 4\tanh(X_{max}) - 3\epsilon sech^4(X_{max}))}{3\,(1 - \tanh\,(X_{max}))\,(4 - 4\tanh(X_0) - 3\epsilon sech^4(X_0))} - \frac{2}{3}} \tag{4.28}$$

It is clear that the soliton oscillates around the well if its initial velocity is lower than the escape velocity. The period of the oscillation can be calculated numerically using equation (4.25).

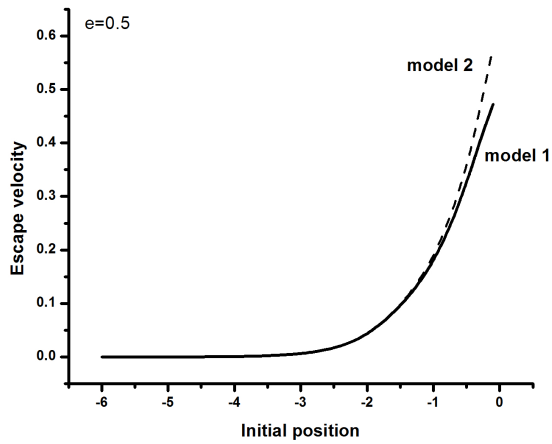

Figure 7: Escape velocity as a function of initial position in both models for $\epsilon = 0.5$.

5 Analytical results versus numerical simulation

It is shown that the general behaviour of a soliton-potential system is almost the same in the models 1 and 2. However, we can find some small differences between the dynamics of a soliton in different models. It is important to compare the results of analytical models with direct numerical solutions. Here we will compare the results of analytic model 1 with its numerical solution. It is clear that the same comparison can be done for the model 2. The soliton equation of motion in the model 1 for a small potential is [15]

$$\phi_{tt} - \phi_{xx} + (1 + \sigma\delta(x))\frac{\partial U}{\partial \phi} = 0 \tag{5.29}$$

Both two models (for a delta-like potential) have a parameter in their equation of motion which controls the strength of the external potential. It is possible to compare the strength parameters in a specific situation by simulation and adjusting parameters to have same results by different models for that specific situation. It is expected to find approximately same relation between the parameters in other situations. We found that the critical velocity for a soliton-barrier system is $v_c = \sqrt{\frac{\epsilon}{2}}$. It is possible to adjust the strength parameter ϵ in analytical model with same parameter in (5.29)

trough the v_c. Numerical simulations using equation (5.29) show the same behaviour for critical velocity. An effective potential can be found by interpolation of simulation results on the $v_c = \sqrt{\frac{\epsilon_{eff}}{2}} = \sqrt{\frac{\alpha+\epsilon\beta}{2}}$. ϵ_{eff} is potential strength (as an effective parameter) in numerical simulation while ϵ is similar parameter in the model 1. ϵ_{eff} can be found by fitting the numerical results on a theoretical diagram as

$$\epsilon_{effective} = (0.0434 \pm 0.01061) + (0.76462 \pm 0.02479)\,\epsilon \tag{5.30}$$

Figure 8 shows the result of simulations of equation (5.29) for the NKG model. Our simulations show very good agreement between numerical results and theoretical predictions for other features of soliton-potential interaction.

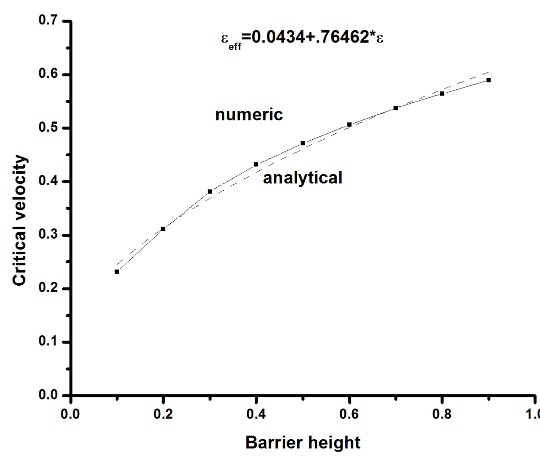

Figure 8: Critical velocity as a function of ϵ with results of simulation using equation (5.29) and analytic model.

6 Conclusion and remarks

Two analytical models for the interaction of the NKG solitons with delta function potential have been presented. Models predict a critical velocity for the soliton-barrier interaction which is a function of initial conditions and the potential identities. For a soliton-well system an escape velocity

was introduced instead of the critical velocity. These models are able to explain most of the features of the system analytically. We have observed that the center of the potential as seen by NKG solitons is a quite different from the real position of the potential center. Numerical simulations are in agreement with theoretical predictions of the models. Our models fail to predict the narrow windows of soliton reflection from the potential well. So, it is expected to find a better model with suitable collective coordinate method to explain this behaviour. These models can be used for prediction of soliton behaviour in the other field theories beside the NKG model.

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